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Extremum of Mahler volume for generalized cylinder in \mathbb{R}^3

Hu Yan

Correspondence: huyan12@126.comDepartment of Mathematics,
Shanghai University of Electric
Power, Shanghai 200090, China**Abstract**

A special case of Mahler volume for the class of symmetric convex bodies in \mathbb{R}^3 is treated here. It is shown that a cube has the minimal Mahler volume and a cylinder has the maximal Mahler volume for all generalized cylinders. Further, the Mahler volume of bodies of revolution obtained by rotating the unit disk in \mathbb{R}^2 is presented.

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1 Introduction

Throughout this article a convex body K in Euclidean n -space \mathbb{R}^n is a compact convex set that contains the origin in its interior. Its polar body K^* is defined by

$$K^* = \{x \in \mathbb{R}^n : x \cdot y \leq 1, \text{ for all } y \in K\},$$

where $x \cdot y$ denotes the standard inner product of x and y in \mathbb{R}^n .

If K is an origin symmetric convex body, then the product

$$V(K)V(K^*)$$

is called the volume product of K , where $V(K)$ denotes n -dimensional volume of K , which is known as the *Mahler volume* of K , and it is invariant under linear transformation.

One of the main questions still open in convex geometric analysis is the problem of finding a sharp lower estimate for the Mahler volume of a convex body K (see the survey article [1]).

A sharp upper estimate of the volume product is provided by the Blaschke-Santaló inequality: For every centered convex body K in \mathbb{R}^n

$$V(K)V(K^*) \leq \omega_n^2,$$

with equality if and only if K is an ellipsoid centered at the origin, where ω_n is the volume of the unit ball in \mathbb{R}^n (see, e.g., [2-5]).

The Mahler conjecture for the class of origin-symmetric bodies is that:

$$V(K)V(K^*) \geq \frac{4^n}{n!} \quad (1.1)$$

with equality holding for parallelepipeds and their polars. For $n = 2$, the inequality is proved by Mahler himself [6], and in 1986, Reisner [7] showed that parallelograms are the only minimizers. Reisner [8] established inequality (1.1) for a class of bodies that have a high degree of symmetry, known as zonoids, which are limits of finite Minkowski sums of line segments. Lopez and Reisner [9] proved the inequality (1.1) for $n \leq 8$ and the minimal bodies are characterized. Recently, Nazarov et al. [10] proved that the cube is a strict local minimizer for the Mahler volume in the class of origin-symmetric convex bodies endowed with the Banach-Mazur distance.

Bourgain and Milman [11] have proved that there exists a constant $c > 0$ independent of the dimension n , such that for all origin-symmetric bodies K ,

$$V(K)V(K^*) \geq c^n \omega_n^2,$$

which is now known as the reverse Santaló inequality. Recently, Kuperberg [12] found a beautiful new approach to the reverse Santaló inequality. What's especially remarkable about Kuperberg's inequality is that it provides an explicit value for c . However, the Mahler conjecture is still open even in the three-dimensional case, Tao [13] made an excellent remark about the open question.

In the present article, we treat a special case of Mahler volume in \mathbb{R}^3 . We now introduce some notations: A real-valued function $f(x)$ is called *concave*, if for any $x, y \in [a, b]$ and any $\lambda \in [0, 1]$, they have

$$f(\lambda x + (1 - \lambda)y) \geq \lambda f(x) + (1 - \lambda)f(y).$$

Definition 1 In three-dimensional Cartesian coordinate system $OXYZ$, if C' is an origin-symmetric convex body in coordinate plane YOZ , then the set:

$$C = \{(x, y, z) \mid -1 \leq x \leq 1, (0, y, z) \in C'\} \quad (1.2)$$

is defined as a generalized cylinder in \mathbb{R}^3 .

Definition 2 In the coordinate plane XOY , let

$$D = \{(x, y) \mid -a \leq x \leq a, |y| \leq f(x)\}, \quad (1.3)$$

where $f(x)$ ($[-a, a]$, $a > 0$), is a nonnegative concave and even function. Rotating D about the X -axis in \mathbb{R}^3 , we can get a geometric object

$$R = \{(x, y, z) \mid -a \leq x \leq a, (y^2 + z^2)^{\frac{1}{2}} \leq f(x)\}. \quad (1.4)$$

We define the geometric object R as a body of revolution generated by the function $f(x)$ (or by the domain D), and call the function $f(x)$ as the generated function of R and D as the generated domain of R .

If the generated domain of R is a rectangle and a diamond, R is called a *cylinder* and a *bicone*, respectively.

Let \mathcal{C} denotes the set of all generalized cylinders. In this article, we proved that among the generalized cylinders, a cube has the minimal Mahler volume and a cylinder has the maximal Mahler volume, theorem as following:

Theorem 1 For $C \in \mathcal{C}$, we have

$$V(C_0)V(C_0^*) \leq V(C)V(C^*) \leq V(C_1)V(C_1^*), \quad (1.5)$$

where $C_0 = [-1, 1] \times [-1, 1] \times [-1, 1]$ is a cube and $C_1 = [-1, 1] \times B^2$ is cylinder.

Further, we get the following theorem:

Theorem 2 For a class of bodies of revolution obtained by rotating the “unit disk” in planar XOY, where the “unit disk” is the following set:

$$U = \{(x, y) \mid |x|^p + |y|^p \leq 1, p \geq 1, \quad (1.6)$$

the Mahler volume is increasing for $1 \leq p \leq 2$ and decreasing for $2 \leq p \leq +\infty$.

More interrelated notations, definitions, and their background materials are exhibited in the following section.

2 Definition and notation

The setting for this article is n -dimensional Euclidean space \mathbb{R}^n . Let \mathcal{K}^n denotes the set of convex bodies (compact, convex subsets with non-empty interiors), \mathcal{K}_o^n denotes the subset of \mathcal{K}^n that contains the origin in their interiors. As usual, B^n denotes the unit ball centered at the origin, S^{n-1} the unit sphere, o the origin, and $\|\cdot\|$ the norm in \mathbb{R}^n .

If $u \in S^{n-1}$ is a direction, u^\perp is the $(n - 1)$ -dimensional subspace orthogonal to u . For $x, y \in \mathbb{R}^n$, $x \cdot y$ is the inner product of x and y , and $[x, y]$ denotes the line segment with endpoints x and y .

If K is a set, ∂K is its boundary, $\text{int } K$ is its interior, and $\text{conv } K$ denotes its convex hull. $V(K)$ denotes n -dimensional volume of K . Let $K|S$ be the orthogonal projection of K into a subspace S .

Let $K \in \mathcal{K}^n$ and $H = \{x \in \mathbb{R}^n \mid x \cdot v = d\}$ denotes a hyperplane, H^+ and H^- denote the two closed halfspaces bounded by H .

Associated with each convex body K in \mathbb{R}^n , its *support function* $h_K : \mathbb{R}^n \rightarrow [0, \infty)$, is defined for $x \in \mathbb{R}^n$, by

$$h_K(x) = \max\{x \cdot y : y \in K\},$$

and its *radial function* $\rho_K : \mathbb{R}^n \setminus \{0\} \rightarrow (0, \infty)$, is defined for $x \neq 0$, by

$$\rho_K(x) = \max\{\lambda \geq 0 : \lambda x \in K\}.$$

From the definitions of the support and radial functions and the definition of the polar bodies, it follows that (see [4])

$$\begin{aligned} h_{K^*}(u) &= \frac{1}{\rho_K(u)} \quad \text{and} \quad \rho_{K^*}(u) = \frac{1}{h_K(u)}, \quad u \in S^{n-1}, \\ K^* &= \{x \in \mathbb{R}^n : h_K(x) \leq 1\}, \\ K^{**} &= K. \end{aligned}$$

If P is a polytope, i.e., $P = \text{conv}\{p_1, \dots, p_m\}$, where p_i ($i = 1, \dots, m$) are vertices of polytope P . By the definition of polar body, we have

$$\begin{aligned} P^* &= \{x \in \mathbb{R}^n : x \cdot p_1 \leq 1, \dots, x \cdot p_m \leq 1\} \\ &= \bigcap_{i=1}^m \{x \in \mathbb{R}^n : x \cdot p_i \leq 1\}, \end{aligned}$$

which implies that P^* is the intersection of m closed halfspace with exterior normal vector p_i and the distance of hyperplane $\{x \in \mathbb{R}^n : x \cdot p_i = 1\}$ from the origin is $1/\|p_i\|$.

For $K \in \mathcal{K}_o^n$, if $x = (x_1, x_2, \dots, x_n) \in K$, $x' = (\varepsilon_1 x_1, \dots, \varepsilon_n x_n) \in K$ for any signs $\varepsilon_i = \pm 1$ ($i = 1, \dots, n$), then K is a *1-unconditional convex body*. In fact, K is symmetric around all coordinate hyperplanes.

To prove the inequality, we give the following definitions.

Definition 3 In Definition 2, if the function

$$f(x) = kx + b, \quad x \in [-a, 0],$$

where k and b are real constants, and $f(-a) = 0$, then the body of revolution is defined as a *bicone*. In three-dimensional Cartesian coordinate system $OXYZ$, if C' is an origin-symmetric convex body in coordinate plane YOZ and points $A = (-a, 0, 0)$ and $A' = (a, 0, 0)$, then the set.

$$B = \text{conv}\{C', A, A'\} \quad (2.1)$$

is defined as a *generalized bicone* in \mathbb{R}^3 .

3 Proof of the main results

In this section, we only consider convex bodies in three-dimensional Cartesian coordinate system with origin O , and its three coordinate axes are denoted by X -, Y -, and Z -axis.

Let C be a generalized cylinder as following:

$$C = \{(x, y, z) \mid -1 \leq x \leq 1, (0, y, z) \in C'\},$$

where C' is an origin-symmetric convex body in coordinate plane YOZ .

We require the following lemmas to prove our main result.

Lemma 1 If $K \in \mathcal{K}_o^3$ for any $u \in S^2$, then

$$K^* \cap u^\perp = (K|u^\perp)^*. \quad (3.1)$$

On the other hand, if $K' \in \mathcal{K}_o^3$ satisfies

$$K' \cap u^\perp = (K|u^\perp)^*,$$

for any $u \in S^2 \cap v_0^\perp$ (v_0 is a fixed vector), then

$$K' = K^*. \quad (3.2)$$

Proof First, we prove (3.1).

Let $x \in u^\perp$, $y \in K$ and $y' = y|u^\perp$, since the hyperplane u^\perp is orthogonal to the vector $y - y'$, then

$$y \cdot x = (y' + y - y') \cdot x = y' \cdot x + (y - y') \cdot x = y' \cdot x.$$

If $x \in K^* \mid u^\perp$, for any $y' \in K \mid u^\perp$, there exists $y \in K$ such that $y' = y|u^\perp$, then $x \cdot y' = x \cdot y \leq 1$, and $x \in (K|u^\perp)^*$. Hence,

$$K^* \cap u^\perp \subseteq (K|u^\perp)^*.$$

If $x \in (K|u^\perp)^*$, then for any $y \in K$ and $y' = y|u^\perp$, $x \cdot y = x \cdot y' \leq 1$, thus $x \in K^*$, and since $x \in u^\perp$, thus $x \in K^* \mid u^\perp$. Then,

$$(K|u^\perp)^* \subseteq K^* \cap u^\perp.$$

Next, we prove (3.2).

Let $S^1 = S^2 \cap v_0^\perp$. For any direction vector $v \in S^2$, there always exists a $u \in S^1$ satisfying $v \in u^\perp$. Since

$$K' \cap u^\perp = (K|u^\perp)^*,$$

and by (3.1)

$$K^* \cap u^\perp = (K|u^\perp)^*,$$

thus

$$K' \cap u^\perp = K^* \cap u^\perp.$$

Then, we get

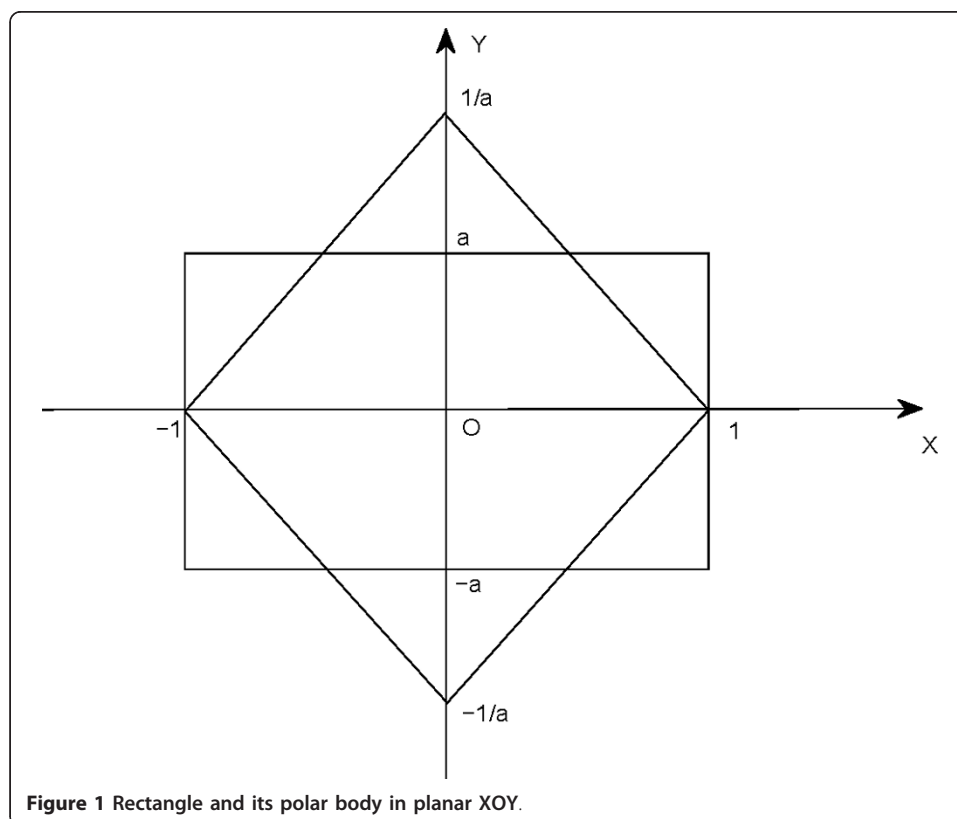
$$\rho_{K'}(v) = \rho_{K^*}(v).$$

By the arbitrary of direction v , we obtain the desired result. ■

For any $C \in \mathcal{C}$ and any $u \in B^2 \mid v^\perp$ ($v = (1, 0, 0)$), $C|u^\perp$ is a rectangle by the above definition. We study the polar body of a rectangle in the planar. From Figure 1, if $C|u^\perp = [-1, 1] \times [-a, a]$, its polar body in planar XOY is a diamond (vertices are $(-1, 0)$, $(1, 0)$, $(0, -1/a)$, $(0, 1/a)$), thus we can get the following Lemma 2.

Lemma 2 For any $C \in \mathcal{C}$, if

$$C = \{(x, y, z) \mid -1 \leq x \leq 1, (0, y, z) \in C'\},$$



where C' is an origin-symmetric convex body in coordinate plane YOZ , then C^* is a generalized bicone with vertices $(-1, 0, 0)$ and $(1, 0, 0)$ and the base $(C')^*$.

Proof Let $v_0 = (1, 0, 0)$, $S^1 = S^2 \cap v_0^\perp$. By Lemma 1, we have

$$C^* \cap u^\perp = (C|u^\perp)^*$$

for any $u \in S^1$. Because that $(C|u^\perp)^*$ is a diamond with vertices $(-1, 0, 0)$ and $(1, 0, 0)$, $C^* \cap u^\perp$ is a diamond with vertices $(-1, 0, 0)$ and $(1, 0, 0)$ for any $u \in S^1$, which implies that C^* is a bicone with vertices $(-1, 0, 0)$ and $(1, 0, 0)$.

In view of

$$C^* \cap v_0^\perp = (C|v_0^\perp)^*$$

and $C|v_0^\perp = C'$, then, the base of C^* is $(C')^*$. ■

In the following, we will restate and prove Theorem 1.

Theorem 1 For $C \in \mathcal{C}$, we have

$$V(C_0)V(C_0^*) \leq V(C)V(C^*) \leq V(C_1)V(C_1^*), \quad (3.3)$$

where $C_0 = [-1, 1] \times [-1, 1] \times [-1, 1]$ is a cube and $C_1 = [-1, 1] \times B^2$ is cylinder.

Proof Let $v = (1, 0, 0)$, and $V(C) = V(C_0)$ by linear transformation, thus $V(C \cap v^\perp) = V(C_0 \cap v^\perp)$.

In planar v^\perp , since the square has the minimal Mahler volume in \mathbb{R}^2 , thus

$$V(C_0 \cap v^\perp)V((C_0 \cap v^\perp)^*) \leq V(C \cap v^\perp)V((C \cap v^\perp)^*),$$

we get

$$V((C_0 \cap v^\perp)^*) \leq V((C \cap v^\perp)^*),$$

then

$$\begin{aligned} V(C_0^*) &= \frac{1}{3}V((C_0 \cap v^\perp)^*) \times 2 \\ &\leq \frac{1}{3}V((C \cap v^\perp)^*) \times 2 \\ &= V(C^*), \end{aligned}$$

where the equality holds if and only if $C \cap v^\perp$ is a square. Hence,

$$V(C_0)V(C_0^*) \leq V(C)V(C^*).$$

Similarly, let $V(C) = V(C_1)$ for any $C \in \mathcal{C}$ by linear transformation, then $V(C \cap v^\perp) = V(C_1 \cap v^\perp)$.

Since $C_1 \cap v^\perp$ is a disk, which has the maximal Mahler volume in \mathbb{R}^2 , thus

$$V(C_1 \cap v^\perp)V((C_1 \cap v^\perp)^*) \geq V(C \cap v^\perp)V((C \cap v^\perp)^*),$$

we get

$$V((C_1 \cap v^\perp)^*) \geq V((C \cap v^\perp)^*).$$

Hence, $V(C_1^*) \geq V(C^*)$, which implies

$$V(C_1)V(C_1^*) \geq V(C)V(C^*).$$

■

Theorem 1 implies that among the generalized cylinders, a cube has the minimal Mahler volume and a cylinder has the maximal Mahler volume.

4 Mahler volume of a special class of bodies of revolution

In this section, we study a special case in the coordinate plane XOY , and define the “unit disk” in planar XOY as following set:

$$U = \{(x, y) \mid |x|^p + |y|^p \leq 1\}, \quad p \geq 1. \quad (4.1)$$

We need the following lemmas to prove our result.

Lemma 3 *Let P is a 1-unconditional convex body and P^* is its polar body in the coordinate plane XOY . Let R and R' are two bodies of revolution obtained by rotating P and P^* , respectively. Then $R' = R^*$.*

Proof Let $v_0 = \{1, 0, 0\}$ and $S^1 = S^2 \cap v_0^\perp$, for any $u \in S^1$, we have

$$R|u^\perp = R \cap u^\perp.$$

Since $R' \cap u^\perp = (R \cap u^\perp)^*$ for any $u \in S^1$, we get

$$R' \cap u^\perp = (R|u^\perp)^*,$$

for any $u \in S^1$. By Lemma 1, we have $R' = R^*$. ■

Lemma 4 *If*

$$\frac{1}{p} + \frac{1}{q} = 1,$$

then the polar body of

$$U = \{(x, y) \mid |x|^p + |y|^p \leq 1\}, \quad p \geq 1$$

is the following set:

$$U' = \{(x, y) \mid |x|^q + |y|^q \leq 1\}, \quad q \geq 1. \quad (4.2)$$

Proof For any $(x, y) \in U$ and $(x', y') \in U'$, we have

$$xx' + yy' \leq |xx'| + |yy'| \leq (|x|^p + |y|^p)^{\frac{1}{p}} (|x'|^q + |y'|^q)^{\frac{1}{q}} \leq 1,$$

which implies $U' \subset U^*$.

If a point $A' = (x', y') \notin U'$, then

$$|x'|^q + |y'|^q > 1.$$

Let $A'_0 = (|x'|, |y'|)$, then $A'_0 \notin U'$. There exists a real $r > 1$ and a point $A^0 \in \partial U'$ satisfying $A'_0 = rA^0$. If $A^0 = (x_0, y_0)$, then $x_0 > 0$ and $y_0 > 0$. Let $x = x_0^{\frac{q}{p}}$ and $y = y_0^{\frac{q}{p}}$, then

$$x^p + y^p = x_0^q + y_0^q = 1$$

and

$$xx_0 + yy_0 = x_0^{1+q/p} + y_0^{1+q/p} = x_0^q + y_0^q = 1,$$

which implies $(x, y) \in U$ and $\langle (x, y), (|x'|, |y'|) \rangle = r > 1$, thus $A'_0 \notin U^*$. Because that U^* is a 1-unconditional convex body, we have $A' \notin U^*$. Then, $U^* \subset U'$. ■

Rotating U and U' , we can get two bodies of revolution R and R' . By Lemma 3, we have $R' = R^*$. Let $F(p) = V(R)V(R^*)$.

In the following, we restate and prove Theorem 2.

Theorem 2 *For a class of bodies of revolution obtained by rotating the “unit disk” in planar XOY, where the “unit disk” is the following set:*

$$U = \{(x, y) \mid |x|^p + |y|^p \leq 1\}, \quad p \geq 1, \quad (4.3)$$

the Mahler volume is increasing for $1 \leq p \leq 2$ and decreasing for $2 \leq p \leq +\infty$.

Proof By integration, we get $V_R(p)$ and $V_{R^*}(q)$, which are volume functions of R and R^* about p and q as following:

$$V_R(p) = 2\pi \int_0^1 (1 - x^p)^{\frac{2}{p}} dx, \quad p \geq 1,$$

and

$$V_{R^*}(q) = 2\pi \int_0^1 (1 - x^q)^{\frac{2}{q}} dx, \quad \frac{1}{p} + \frac{1}{q} = 1.$$

Thus, we have the Mahler volume $V(R)V(R^*)$, which is a function about p as following:

$$F(p) = V_R(p)V_{R^*}(q) = 4\pi^2 \int_0^1 (1 - x^p)^{\frac{2}{p}} dx \int_0^1 (1 - x^q)^{\frac{2}{q}} dx, \quad (4.4)$$

where $p \geq 1$, and $\frac{1}{p} + \frac{1}{q} = 1$.

Let $1 - x^p = y$, we have

$$\int_0^1 (1 - x^p)^{\frac{2}{p}} dx = \frac{1}{p} \int_0^1 y^{\frac{2}{p}} (1 - y)^{\frac{1}{p}-1} dy = \frac{1}{p} B\left(\frac{2}{p} + 1, \frac{1}{p}\right),$$

where $B(\cdot, \cdot)$ is Beta function. Thus we have

$$F(p) = \frac{4\pi^2}{pq} B\left(\frac{2}{p} + 1, \frac{1}{p}\right) B\left(\frac{2}{q} + 1, \frac{1}{q}\right),$$

where $p \geq 1$, and $\frac{1}{p} + \frac{1}{q} = 1$.

By the relationship between Gamma function and Beta function:

$$B(x, y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)},$$

we have

$$F(p) = \frac{4\pi^2}{pq} \cdot \frac{\Gamma(\frac{2}{p} + 1)\Gamma(\frac{1}{p})}{\Gamma(\frac{3}{p} + 1)} \cdot \frac{\Gamma(\frac{2}{q} + 1)\Gamma(\frac{1}{q})}{\Gamma(\frac{3}{q} + 1)}.$$

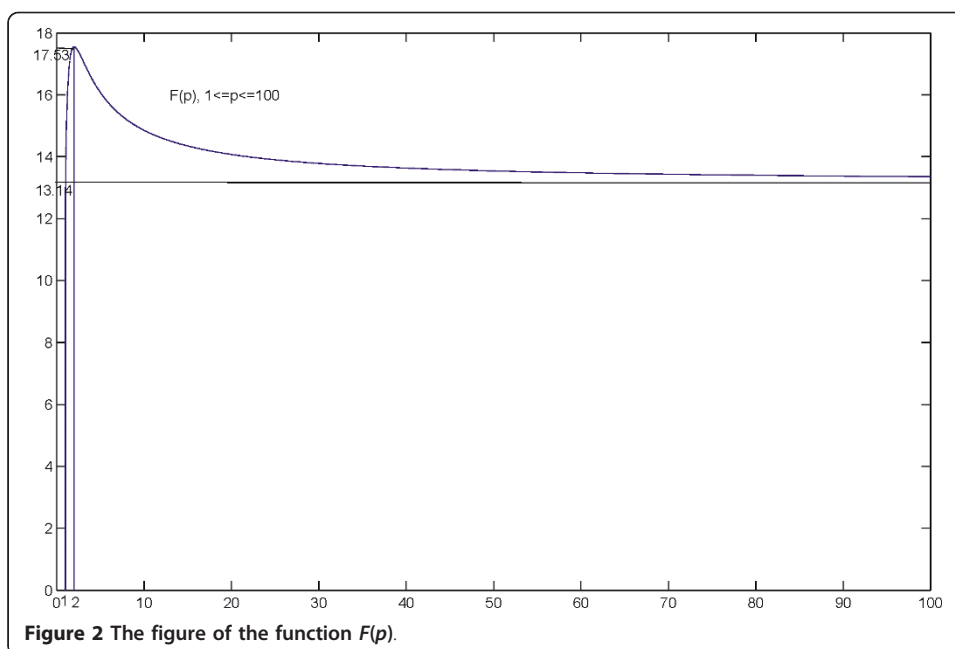


Figure 2 The figure of the function $F(p)$.

And by the following properties of Gamma function:

$$\Gamma(z+1) = z\Gamma(z) \text{ and } \Gamma(1-z)\Gamma(z) = \frac{\pi}{\sin(\pi z)},$$

we have

$$F(p) = \frac{16\pi^3}{9} \cdot \frac{(p-1)(p-2)}{p(2p-3)(p-3)} \cdot \frac{\sin(\frac{3\pi}{p})}{\sin(\frac{2\pi}{p})\sin(\frac{\pi}{p})}, \quad p \geq 1.$$

We can easily prove

$$\lim_{p \rightarrow 1} F(p) = \lim_{p \rightarrow +\infty} F(p) = \frac{4\pi^2}{3}, \quad (4.5)$$

then R and R^* are bicone and cylinder, or cylinder and bicone, and

$$\lim_{p \rightarrow 2} F(p) = \frac{16\pi^2}{9}, \quad (4.6)$$

then R and R^* are the same unit ball, which have the maximal Mahler volume.

In fact, $F(p) = F(q)$ holds when $\frac{1}{p} + \frac{1}{q} = 1$, so we just need to prove $F(p)$ is increasing when $1 \leq p \leq 2$, which can be easily proved by $F'(p) \geq 0$ when $1 \leq p \leq 2$. Based on the above conclusions, we have that a cylinder has the minimal Mahler volume and a ball has the maximal Mahler volume in this special class of bodies of revolution. ■

We can draw the figure of the function $F(p)$ by using MATLAB (see Figure 2). From the figure, we see that function $F(p)$ is increasing when $1 \leq p \leq 2$ and decreasing when $2 \leq p \leq +\infty$, so $F(2)$ is a maximum and $F(1) = F(+\infty)$ is a minimum.

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Competing interests

The author declares that they have no competing interests.

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